Int. J. Solids Structures Vol. 28, No. 4, pp. 403–412, 1991
Printed in Great Britain.

0020-7683/91 $3.00 + .00

ON THE PROPERTIES OF PLASTIC FLOW OF MATERIAL UNDER NONPROPORTIONAL CYCLIC LOADING

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(*Received* 28 *March* 1989; *in revised form* 25 *September* 1990)

Abstract—A series of experiments on TB451-63 axle steel were conducted for the investigation of (1) the basic material plastic flow properties under plastic nonproportional cyclic loading, and (2) the influence of the deformation path, history and direction on the material hardening behavior and plastic flow properties. Comparative studies of the present experimental results with the theoretical predictions of the endochronic constitutive theory reveal the inadequacy of uniaxial test data in describing the basic properties of materials under nonproportional cyclic loading. Thus, to achieve adequacy and applicability for the description of nonproportional cyclic constitutive behavior of materials, modifications to the endochronic constitutive theory are absolutely necessary.

INTRODUCTION

One significant problem for researchers in scientific investigations and engineering applications, due to the rapid development of modern industry, stems from the increasingly severe loading and environmental conditions to which many structures such as aircraft, turbine and rocket engines, nuclear reactor components and so forth are subjected. Therefore there is an urgent need for a constitutive theory of materials, which aims to provide adequate, applicable and accurate descriptions of constitutive behavior, damage evolution, and fatigue and failure behavior of materials under the aforementioned conditions. In the past two decades, both the theoretical and experimental aspects of the uniaxial constitutive behavior of materials have been systematically investigated by various researchers. Unfortunately, nearly all the material constitutive models formulated thus far appear incapable of describing the nonproportional cyclic deformation behavior of materials, although they proved successful in cases when uniaxial tensile cyclic and creep behavior were dealt with. This is attributable to the fact that uniaxial experiments are by no means able to reveal the basic deformation behavior characteristics of materials when subjected to nonproportional cyclic loading environments. Indeed, we are strikingly ignorant of the constitutive behavior of materials under such complex loading conditions (Lamba and Sidebottom, 1978a,b). It is clearly important to conduct systematic experimental investigations exploring the basic properties of materials under nonproportional cyclic loading, as a pre-requisite to formulating a material constitutive model for nonproportional cycles. However, much remains to be done before such an adequate and applicable model can be engendered (Kremple and Lu, 1984; Tanaka *et al.*, 1985; McDowell, 1985; Benallal and Marquis, 1987).

An insight into the plastic flow properties of materials in nonproportional cycles plays the most important role in the formulation of an accurate constitutive model of materials as long as it is the *de facto* starting point; which is the conceptual basis for the formulative methodology of Ohashi *et al.* (1985) in their investigation of the plastic flow properties of 316 stainless steel under a square strain path. Owing to the fact that the nonproportional cyclic constitutive behavior of materials is strongly dependent on the strain path, a dilemma yet to be overcome is that caused by the lack of systematic knowledge of the basic material properties concerning plastic flow under nonproportional cyclic loading, such as the effects of the shape of the deformation path, the loading direction and the deformation history.

In recent years, the endochronic constitutive theory proposed by Valanis (1980) has received widespread attention due to its attractive capability of accurately describing the constitutive behavior of materials under certain types of loading conditions. The theory is primarily formulated on the basis of the hypothesis that the current stress state of materials can be represented in terms of a functional concerning the entire deformation history defined by the "intrinsic time scale", implying material memory. In a theoretical analysis concerning the plastic flow properties of materials predicted by endochronic constitutive theory and conventional plasticity theory, Murakami and Read (1987) indicated the substantial difference of that theory from its counterparts.

Based on a series of experiments at room temperature, the present work is mainly focused on the investigation of the hardening behavior and some of the basic plastic flow properties of TB451-63 axle steel under nonproportional cyclic loading. Specifically, for these properties, the influence of strain paths, loading direction and previous history is dealt with in parallel with Lensky's hypothesis of local determinability. It should be noted that the hardening behavior and plastic flow properties of materials are strongly dependent on both the shape of the strain path and the deformation history. Comparison between predictions from the endochronic constitutive theory and experimental results further confirms the necessity to have this theory modified by describing the nonproportional cyclic constitutive behavior of materials.

EXPERIMENTAL PROCEDURE

The specimen material is TB451-63 axle steel whose chemical composition is: C: 0.37, Mn: 0.48, Si: <0.15, P: <0.045, S: <0.05, Cr: <0.3, Cu: <0.3, Ni: <0.2, in weight percentage. The geometry of the specimen is given in Fig. 1. Experiments were conducted on a computer-controlled testing system comprising a DEC PDP 11/23 processor, a MTS 809 servo-controlled electro-hydraulic testing machine and a data acquisition system. Strain components in the specimen were evaluated using the measurements from a set of three strain gauges bonded on the outer surface of the specimen. To minimize the viscosity effects, the experiments were run at a low strain rate of 7×10^{-5} s^{-1} and at room temperature.

EXPERIMENTAL RESULTS AND DISCUSSIONS

1. *Stress and strain deviatoric vector space*

The definition of the axial torsional subspace follows as a subspace of Ilyushin's five-dimensional deviatoric vector space. Define the stress vector as

$$\boldsymbol{\sigma} = \sigma_1 \mathbf{n}_1 + \sigma_3 \mathbf{n}_3 \tag{1}$$

where $\sigma_1 = \sigma_{zz} = \sigma$, $\sigma_3 = \sqrt{3}\tau$, and σ and τ are components of the axial stress and shear stress, respectively. $\mathbf{n}_1$ and $\mathbf{n}_3$ are orthonormal base vectors in the stress space.

The strain vector is defined as

$$\boldsymbol{\varepsilon} = \varepsilon_1 \mathbf{n}_1 + \varepsilon_3 \mathbf{n}_3 \tag{2}$$

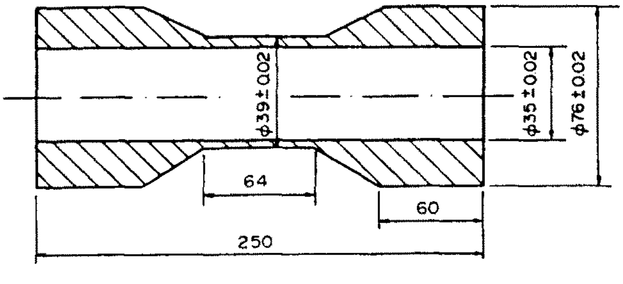


Fig. 1. Shape and geometrical sizes of specimen.

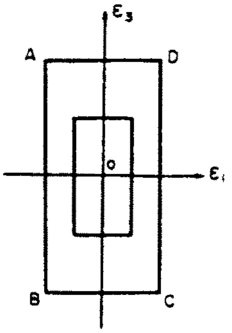


Fig. 2. Rectangular strain path.

where $\varepsilon_1 = \varepsilon_{zz} = \varepsilon$, $\varepsilon_3 = \gamma/\sqrt{3}$, and ε and γ are components of axial strain and engineering shear strain, respectively.

The von Mises equivalent stress may be expressed as

$$\sigma_e = |\boldsymbol{\sigma}| = \{(\sigma_1)^2 + (\sigma_3)^2\}^{1/2}. \tag{3}$$

The equivalent strain is expressed as

$$\varepsilon_e = |\boldsymbol{\varepsilon}| = \{(\varepsilon_1)^2 + (\varepsilon_3)^2\}^{1/2}. \tag{4}$$

the strain rate vector is defined as:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\varepsilon}_1 \mathbf{n}_1 + \dot{\varepsilon}_3 \mathbf{n}_3 \tag{5}$$

and the length of strain path is defined as:

$$S = \int_0^t |\mathrm{d}\boldsymbol{\varepsilon}/\mathrm{d}t|\, \mathrm{d}t. \tag{6}$$

2. *Deformation path and loading path*

Out-of-phase experiments on rectangular strain paths were conducted for two purposes: (1) to study the plastic flow properties of the material after imposing a 90° abrupt change of deformation direction on the rectangular strain path (Fig. 2), (2) to investigate the effects of the previous deformation history. Note that in these experiments, the strain ranges are $\Delta\varepsilon_1/2 = 0.15\%$, $\Delta\varepsilon_3/2 = 0.3\%$ and $\Delta\varepsilon_1/2 = 0.3\%$, $\Delta\varepsilon_3/2 = 0.6\%$, respectively.

To exclude the influence of the previous deformation history, so that one can concentrate on investigating the effects of deformation direction, experiments on rhombic strain trajectories (Fig. 3) were performed with strain ranges $\Delta\varepsilon_1/2 = 0.21\%$, $\Delta\varepsilon_3/2 = 0.42\%$ and

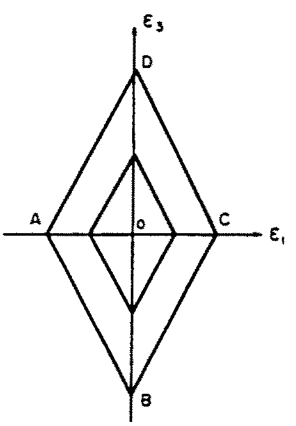


Fig. 3. Rhombic strain path.

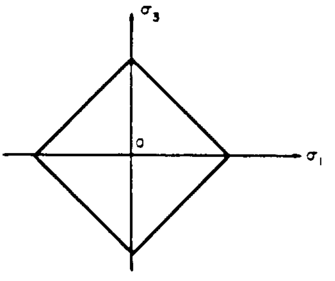


Fig. 4. Stress path.

$\Delta\varepsilon_1/2 = 0.42\%$, $\Delta\varepsilon_3/2 = 0.84\%$, respectively. Note that in these experiments, the change of deformation direction can be either greater or less than 90°.

Experiments on the stress path (Fig. 4) are also conducted on a stress control basis, to study the plastic flow properties of the material for cases when an abrupt change of direction of loading path occurs. The amplitudes of the path involved therein are $\Delta\sigma_1/2 = \Delta\sigma_3/2 = 300$ MPa.

Finally, in investigating the plastic flow properties of the material for a smooth deformation path, out-of-phase experiments on circular and elliptical strain paths were also conducted.

3. *Results and discussions*

It can be seen from Fig. 5 that TB451-63 axle steel is a cyclic stabilized material when subjected to strain-controlled torsional cycles. Also, for a fixed strain range, no cyclic hardening appears. Figure 6 shows rectangular and rhombic strain paths, as shown in Figs 2–3. It is observed that the stress amplitude increases with increasing cyclic number, *N*. Hardening does take place in the nonproportional cyclic deformation of a material even though it is cyclically stabilized in a uniaxial cyclic deformation. This conclusion is compatible with that of Ohashi *et al.* (1985). Comparing Fig. 6a with Fig. 6b naturally comes down to the point that material hardening behavior is immensely dependent upon the shape of the deformation path.

The different stress responses shown in Fig. 6a are also indicated in the vicinity of corner points A and B for a rectangular strain path, although both are on the verge of a sudden change of direction to a 90° veering. As can be seen from Fig. 6a, for the corner

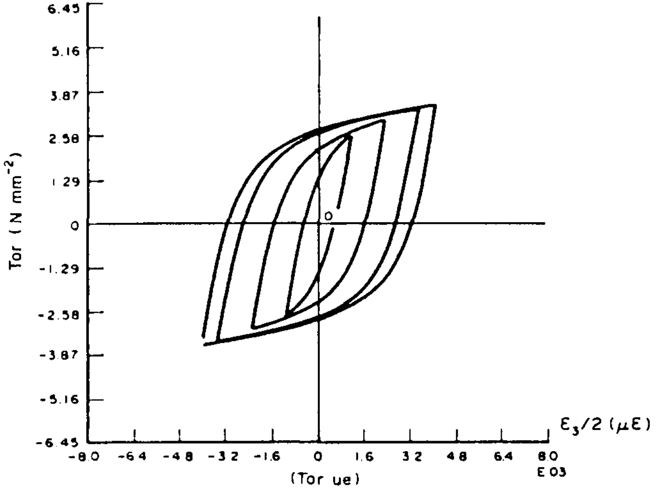


Fig. 5. Stress–strain relations of material under pure cyclic torsion.

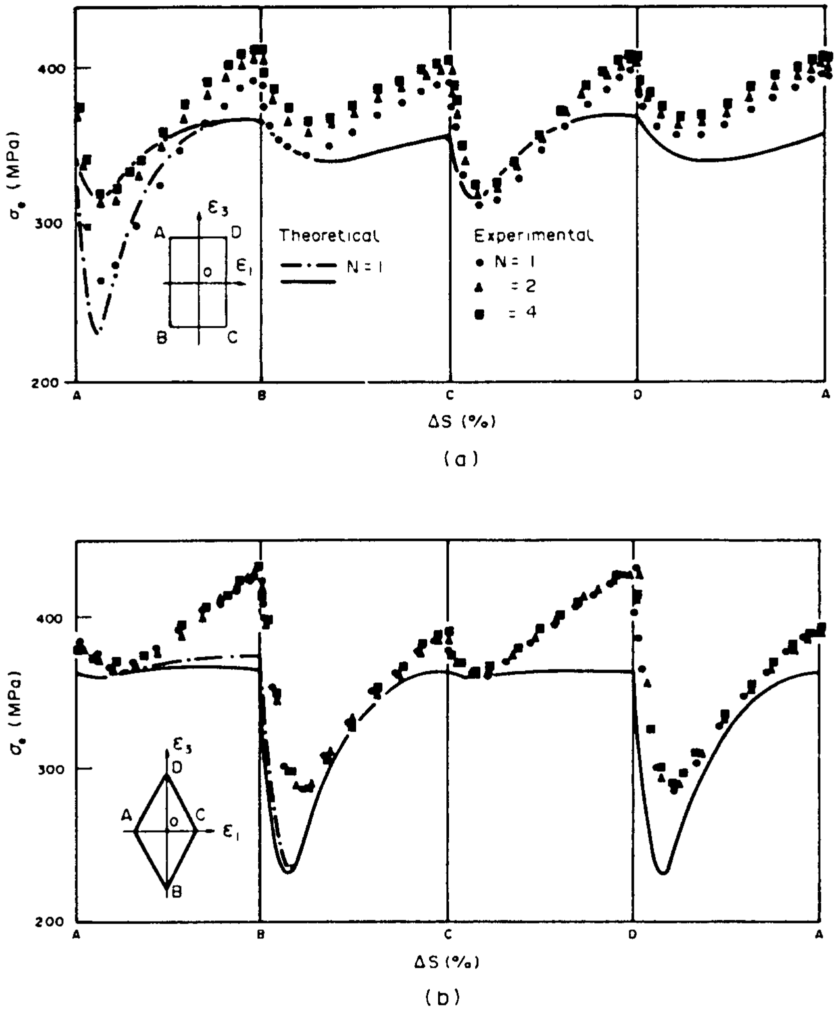
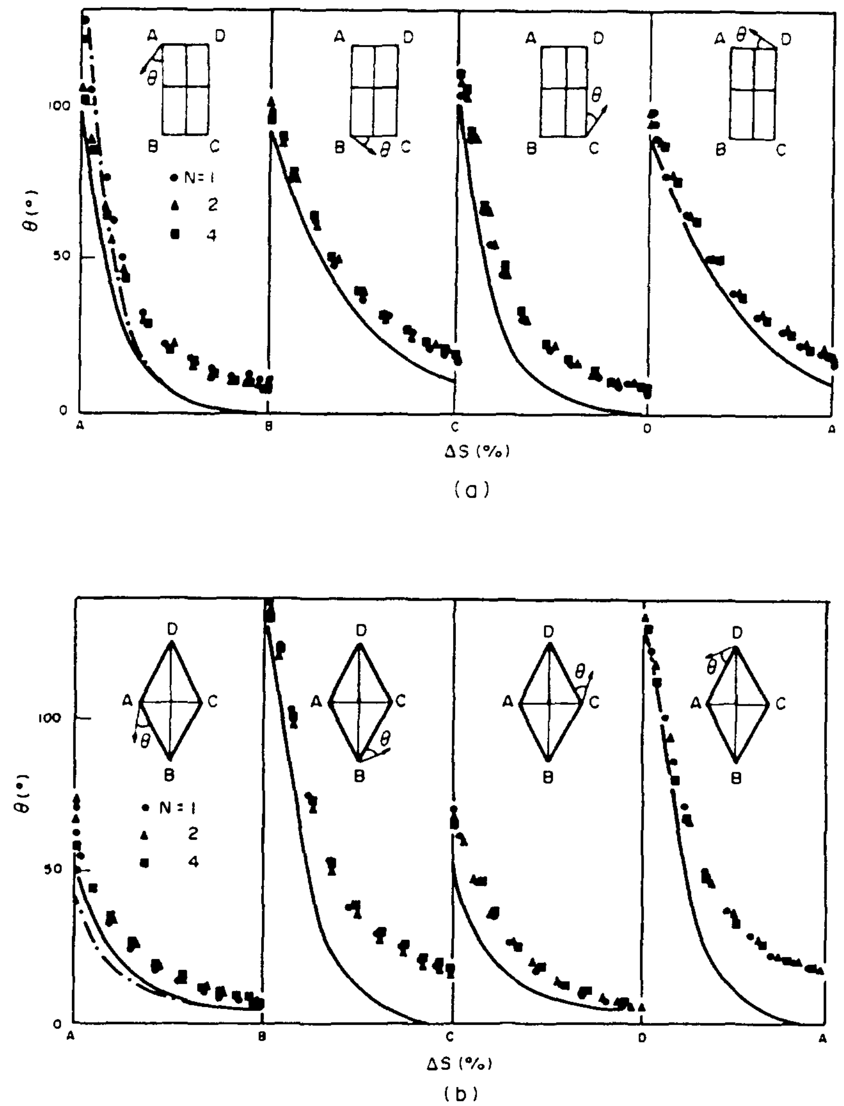


Fig. 6. Variations of stress amplitude with length of strain paths.

point that is approached by the long smooth path, stress relaxation happens immediately after the veering. However, for the corner point with the short approach, 90 veering always implies a post-veering unstable stress response. Since the strain amplitudes at these corner points are of the same value, it is apparent that not only does the material response depend on the current strain amplitude, but it also depends on the deformation history prior to the path veering, in which case there is an abrupt change of direction on the deformation path of 90 .

Considering rhombic strain paths with sides of equal length, Fig. 6b demonstrates the significant dependence of the material responses on the veering angle of the path. Also, less significant stress relaxation is indicated in situations with acute veering angles, and vice versa for obtuse cases.

With regard to the plastic flow behavior of materials under complex loading, Lensky's hypothesis of material local determinability claims that variations of the delay angle θ extended by the strain rate and stress vectors entirely depend on the current value of delay angle θ and the arc length S of the deformation path. In Ohashi *et al.* (1985), when the adequacy and applicability of Lensky's hypothesis were assessed via an out-of-phase experiment using 316 stainless steel with a square strain path, they pointed out the invalidity of the hypothesis under the strain path just mentioned. Figure 7 shows the behavior of the delay angle θ for strain paths specified in Figs 2 and 3 with strain ranges $\Delta\varepsilon_1/2 = 0.3\%$, $\Delta\varepsilon_3/2 = 0.6\%$, and $\Delta\varepsilon_1/2 = 0.42\%$, $\Delta\varepsilon_3/2 = 0.84\%$, respectively. According to Fig. 7a, the



(a)

(b)

Fig. 7. Relations between delay angle θ and length of strain paths.

delay angle θ is exclusively dependent on the arc length of the post-veering portion of the strain path, while the pre-veering portion is totally uninvolved. Figure 8a shows the variation of delay angle θ with the arc length of the strain path measured from corner points A and B. It happens that they are almost located on an identical curve. Hence, Lensky's hypothesis of local determinability is approximately valid for rectangular strain paths. However, for rhombic strain paths, as shown in Fig. 7b, significant difference are indicated for θ-ΔS relations based on acute and obtuse veering angles, where ΔS is the arc length of the post-veering strain path. Specifically, steep variations of θ versus ΔS do occur for cases of obtuse veering angles. In fact, different values of θ can be extrapolated for different veering angle cases from the same ΔS (Fig. 8b), implying the invalidity of Lensky's hypothesis.

Figure 9 shows the variation of delay angle θ with smooth deformation histories under circular and elliptical strain paths. It turns out that θ is almost constant along circular paths, while for elliptical paths, the flatter the ellipse, the sharper the variation of θ. Apparently, Lensky's hypothesis does not hold even on smooth strain paths. Finally, Fig. 10 demonstrates the gradually varying θs from obtuse to acute angles after the loading path abruptly changes its direction. The above-mentioned observations naturally lead to the conclusion that Lensky's hypothesis, in general, is not valid.

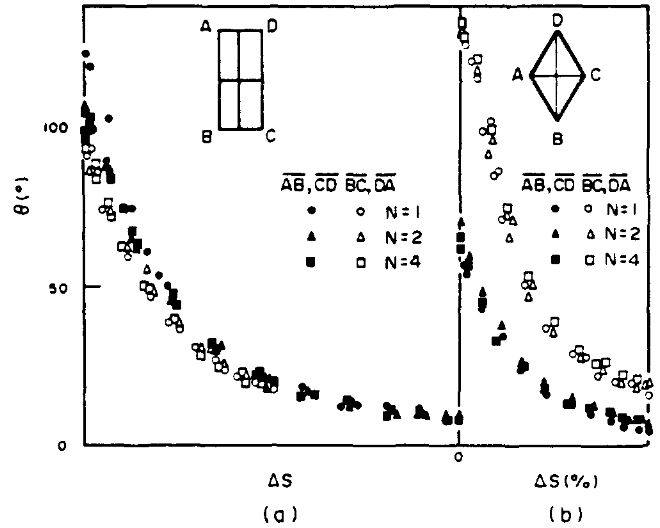


Fig. 8. Relations between delay angle θ and length of strain path after veering.

PREDICTIONS BY ENDOCHRONIC CONSTITUTIVE THEORY

In recent years, endochronic constitutive theory has received wide-spread attention. Formulated on the basis of the irreversible thermodynamics of internal variables, the theory does not have to rely upon the yield surface concept, and the material memory is defined in terms of an intrinsic time scale, a material property at hand. Murakami and Read (1987) investigated on the plastic flow properties conceived in this constitutive model. As indicated in their numerical analysis, the plastic flow rule incorporated in this model is substantially different from that of conventional plasticity theory. It seems that the endochronic constitutive theory is favorably advantageous in providing a straightforward description of material multiaxial constitutive behavior since the model parameters involved are available via uniaxial experiments (Valanis, 1983).

In deviatoric vector space, the basic equation of endochronic theory is

$$\sigma = \int_0^z \rho(z - z')(\mathrm{d}\varepsilon^p/\mathrm{d}z')\,\mathrm{d}z' \tag{7}$$

where

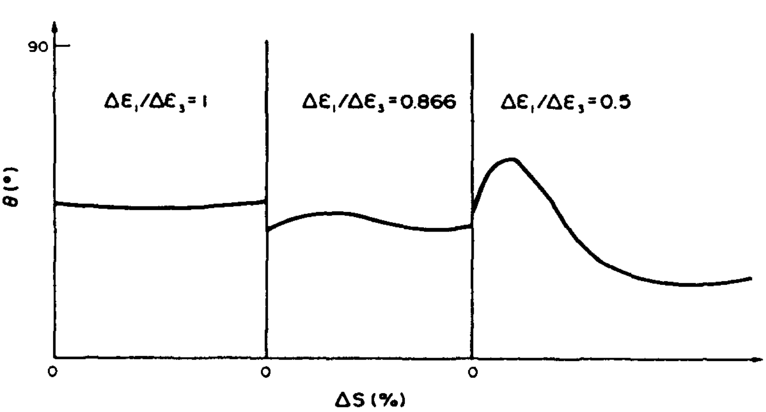


Fig. 9. Variations of delay angle θ with length of circular and elliptical strain paths.

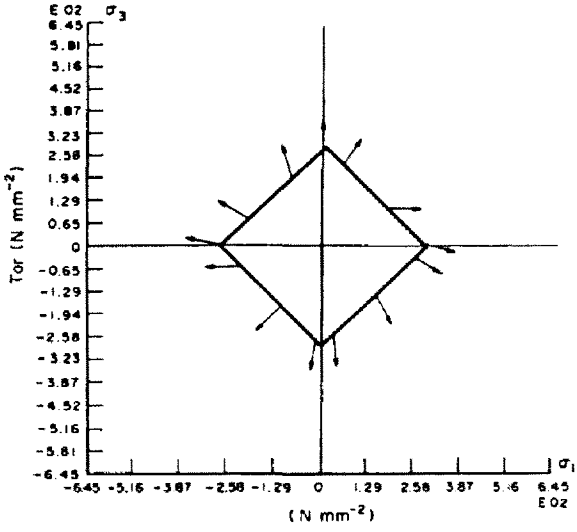


Fig. 10. Variations of strain vector along stress path.

$$\mathrm{d}z = \mathrm{d}\xi / f(\xi) \tag{8}$$

$$\mathrm{d}\xi = \{(\mathrm{d}\varepsilon_1^p)^2 + (\mathrm{d}\varepsilon_3^p)^2\}^{1/2}, \tag{9}$$

$\mathrm{d}\varepsilon_1^p$ and $\mathrm{d}\varepsilon_3^p$ being components of plastic strain increments. As long as TB451-63 axle steel is a cyclic stabilized material with a fixed range of strain (Fig. 5), one may assume that

$$f(\xi) = 1. \tag{10}$$

In the case where the strain history is prescribed, according to Murakami and Read (1987), one has

$$\{[Q_1/(R+E)]^2 + [Q_3/(R+E)]^2 - 1\}\,\mathrm{d}Z^2 + 2[\mathrm{d}\varepsilon_1 Q_1 E/(R+E)^2 + 3\,\mathrm{d}\varepsilon_3 Q_3 G/(R+3G)^2]\,\mathrm{d}Z \\ + [\mathrm{d}\varepsilon_1 E/(R+E)]^2 + [3\,\mathrm{d}\varepsilon_3 G/(R+3G)]^2 = 0 \tag{11}$$

where

$$\mathrm{d}\boldsymbol{\sigma} = R\,\mathrm{d}\boldsymbol{\varepsilon}^p - \mathbf{Q}\,\mathrm{d}Z \tag{12}$$

$$\mathbf{Q} = Q_1\mathbf{n}_1 + Q_3\mathbf{n}_3 = \sum_{\gamma=1}^{3} \alpha_r(Q_{r1}\mathbf{n}_1 + Q_{r3}\mathbf{n}_3) = \sum_{\gamma=1}^{3} \alpha_\gamma \mathbf{Q}_\gamma \tag{13}$$

$$\mathrm{d}\mathbf{Q}_r/\mathrm{d}Z + \alpha_r\mathbf{Q}_r = R_r\,\mathrm{d}\boldsymbol{\varepsilon}^p/\mathrm{d}Z \tag{14}$$

$$R = \sum_{\gamma=1}^{3} R_r \tag{15}$$

$$\mathrm{d}\varepsilon_1^p = \mathrm{d}\varepsilon_1 - \mathrm{d}\sigma_1/E \tag{16}$$

$$\mathrm{d}\varepsilon_3^p = \mathrm{d}\varepsilon_3 - \mathrm{d}\sigma_3/(3G) \tag{17}$$

G and E being the material shear modulus and Young's modulus, respectively. The model parameters α_r and R_r are available from material torsional cyclic experiments as

$$(\alpha_1, \alpha_2, \alpha_3) = (1.54, 0.28, 0.046) \times 10^4$$

$$(R_1, R_2, R_3) = (13.65, 3.57, 0.395) \times 10^2 \text{ GPa}$$

$$E = 200 \text{ GPa}, \quad G = 80 \text{ GPa}.$$

In Figs 6–7, solid curves represent the theoretical predictions of endochronic constitutive theory. As demonstrated in Fig. 6a, for rectangular strain paths (Fig. 2) the endochronic constitutive theory is inadequate to provide a satisfactory description of material hardening behavior, except for the material responses in the vicinity of the corner points of the path. Noting that in the vicinity of the corner points material exhibits elastic unloading, it is apparent that the coincidence between the theoretical prediction and experimental results makes no sense in such a vicinity. In Fig. 6b, the endochronic constitutive theory is not favored for acute veering, albeit for obtuse cases. This is attributable to the fact that a sufficiently extended portion of the post-veering strain path is characterized by the material responses featuring the elastic range for obtuse cases.

Figure 7b supports the adequacy of the endochronic constitutive theory for describing the plastic flow properties of the material in rectangular strain path situations. For the delay angle θ, predictions of the endochronic theory in Fig. 7b contradict those in Fig. 6b. It is apparent from the observations in Fig. 7 that the plastic flow rule incorporated in the endochronic theory is consistent with experimental results only in cases where the material has experienced a sufficiently extended process of plastic deformation.

The predictions of the endochronic theory shown in Figs 6 and 7 prove that this theory cannot accurately describe the hardening behavior and plastic flow properties of the material under nonproportional cyclic loading. This is attributable to the fact that the intrinsic time scale defined in current endochronic constitutive theory is inadequate. It cannot reflect the effects of hardening mechanisms and plastic flow properties of materials under nonproportional cyclic loading. By properly defining the intrinsic time scale of the endochronic constitutive theory, we can improve its predictions for the constitutive behavior of materials under nonproportional cyclic loading (Ning, 1989).

CONCLUDING REMARKS

The experimental investigation of the nonproportional cyclic plasticity of TB451-63 axle steel has resulted in several new observations of material behavior. Certain definite remarks can be made on the basis of this investigation, although the experimental study in this area is by no means complete.

(1) The existence of substantial differences in the plastic flow properties of TB451-63 axle steel under nonproportional cyclic loading from those of the same material under uniaxial and multiaxial proportional cyclic loading is confirmed, the nonproportional cases being closely related to the shape of the deformation path.

(2) The plastic flow properties are significantly influenced by abrupt changes in the direction of deformation path. It is therefore necessary to have this important factor included in the constitutive model for the description of the constitutive behavior of materials under nonproportional cyclic loading situations.

(3) In nonproportional cyclic deformation situations, materials feature remarkable hardening behavior. Uniaxial experimental data are incapable of elucidating the basic properties of materials under nonproportional cyclic loading.

(4) Lensky's hypothesis of local determinability is valid only for some specific deformation paths, but generally this hypothesis does not hold. Not only do the plastic flow properties depend on the current state of the material, but they also depend on its deformation history.

(5) Endochronic constitutive theory is incapable of accurately describing the constitutive behavior of materials under nonproportional cyclic loading, although it does provide good predictions for the plastic behavior corresponding to some specific deformation paths.

It is therefore, in general, necessary to have the endochronic constitutive theory modified to describe the constitutive behavior of materials under nonproportional cyclic loading.

Acknowledgements—We are very grateful to Prof. D. P. Chen for giving us many suggestions and help in the revision of our manuscript. The financial support of the NSFC is also acknowledged.

REFERENCES

Benallal, A. and Marquis, D. (1987). Constitutive equations for nonproportional cyclic elasto-viscoplasticity. *ASME J. Engng Mater. Tech.* **109**, 326–336.

Kremple, E. and Lu, H. (1984). The hardening and rate-dependent behavior of fully annealed AISI type 304 stainless steel under biaxial in-phase and out-of-phase strain cycling at room temperature. *ASME J. Engng Mater. Tech.* **106**, 376–382.

Lamba, H. S. and Sidebottom, O. M. (1978a). Cyclic plasticity for nonproportional paths: part 1—cyclic hardening, erasure of memory, and subsequent strain hardening experiments. *ASME J. Engng Mater. Tech.* **100**, 96–103.

Lamba, H. S. and Sidebottom, O. M. (1978b). Cyclic plasticity for nonproportional paths: part 2—comparison with three incremental plasticity models. *ASME J. Engng Mater. Tech.* **100**, 104–111.

McDowell, D. L. (1985). An experimental study of the structure of constitutive equation for nonproportional cyclic plasticity. *ASME J. Engng Mater. Tech.* **107**, 307–315.

Murakami, H. and Read, H. E. (1987). Endochronic plasticity: some basic properties of plastic flow and failure. *Int. J. Solids Structures* **23**, 133–151.

Ning, J. (1989). Constitutive equations of material under nonproportional cyclic loading. Ph.D. dissertation, Southwestern Jiaotong University, P.R.C.

Ohashi, Y., Tanaka, E. and Ooka, M. (1985). Plastic deformation behavior of type 316 stainless steel subject to out-of-phase strain cycles. *ASME J. Engng Mater. Tech.* **107**, 286–292.

Tanaka, E., Murakami, S. and Ooka, M. (1985). Effects of plastic strain amplitudes on non-proportional cyclic plasticity. *Acta Mech.* **57**, 167–182.

Valanis, K. C. (1980). Fundamental consequences of a new intrinsic time measure: plasticity as a limit of the endochronic theory. *Arch. Mech.* **27**, 171–191.

Valanis, K. C. (1983). Short presentation. In *Theoretical Foundation for Large-scale Computations for Nonlinear Material Behavior* (Edited by S. Nemat-Nasser, R. J. Asaro and G. A. Hegemier), pp. 364–368. Martinus Nijhoff, Dordrecht.